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Bosonic String in Anti-de Sitter Space

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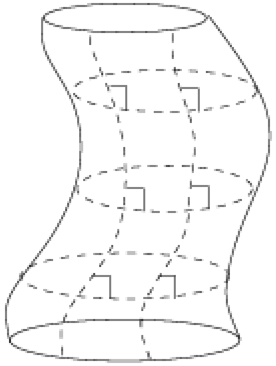
What is string theory?

1st quantisation	2nd quantisation
Path $x : I \longrightarrow \mathcal{M}$	Field $\phi \in \Gamma(E, \mathcal{M})$
$\int \mathcal{D}x \exp(-S[x])$	$\int \mathcal{D}\phi \exp(-S[\phi])$

What is string theory?

General Relativity is a field theory with metric $g_{\mu\nu}$ and action S_{EH}.

It is highly non-linear and non-renormalizable. How to quantise gravity?



1st quantisation + a length l_s

We get worldsheet instead of worldline

Embedding $X\colon Woldsheet \rightarrow Spacetime$

What is string theory?

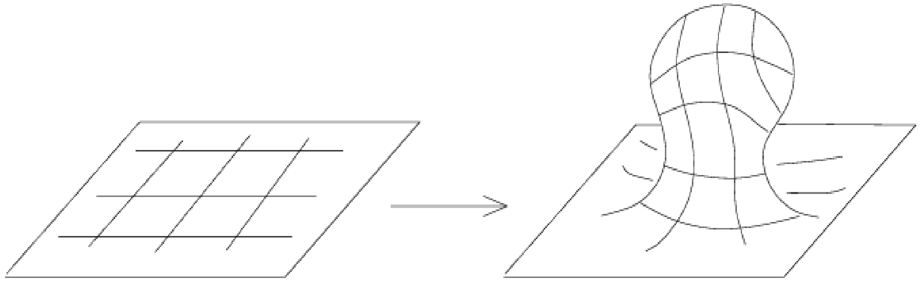
Polyakov action:

$$\mathcal{S} = \frac{T}{2} \int \mathrm{d}^2\sigma \sqrt{-h}\, h^{ab} g_{\mu\nu}(X) \partial_a X^\mu(\sigma) \partial_b X^\nu(\sigma),$$

Symmetries:

- Poincare symmetry in spacetime
- Reparametrization on world sheet
- Weyl symmetry

$$g_{\alpha\beta}(\sigma) \rightarrow \Omega^2(\sigma)\, g_{\alpha\beta}(\sigma)$$



A very constrained system

What is string theory?

Equations of motion and quantize! Promote X to an operator.

Modes satisfy the Virasoro algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n-1)n(n+1)\delta_{n+m,0}\mathbf{1}$$

- 2D field theory are well-understood and rich.
- D=26
- massless spin 2 particle is Einstein gravity. Reduce to GR.

What is Anti-de Sitter Space?

The anti-de Sitter space of signature (p, q) can then be isometrically embedded in the space $\mathbb{R}^{p,q+1}$ with coordinates (x_1, ..., x_p, t_1, ..., t_{q+1}) and the metric

$$ds^2 = \sum_{i=1}^{p} dx_i^2 - \sum_{j=1}^{q+1} dt_j^2$$

as the quasi-sphere

$$\sum_{i=1}^{p} x_i^2 - \sum_{j=1}^{q+1} t_j^2 = -\alpha^2,$$

where α is a nonzero constant with dimensions of length (the radius of curvature). This is a

Lorentzian manifold with constant negative curvature.
Universe with negative cosmological constant.

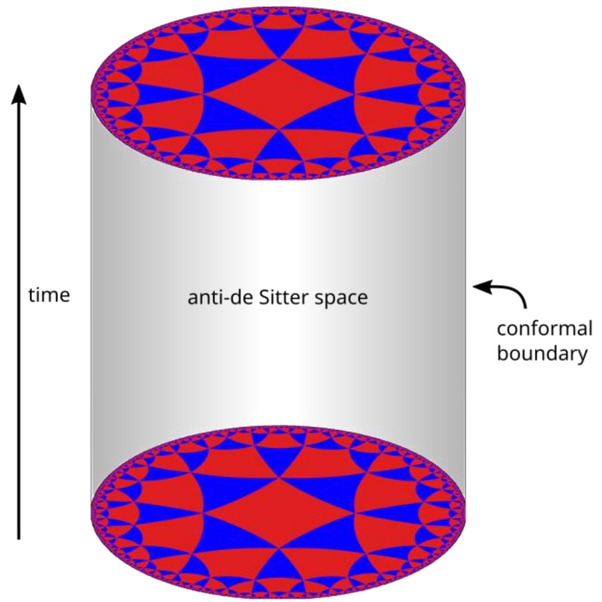
3-dimensional Anti-de Sitter Space

3 dimensional anti-de Sitter space:

$$X_{-1}^2 + X_0^2 - X_1^2 - X_2^2 = 1.$$

Consider Matrices paramterised by:

$$g = \begin{pmatrix} X_{-1} + X_1 & X_0 - X_2 \\ -X_0 - X_2 & X_{-1} - X_1 \end{pmatrix},$$



Set determinant to 1, the hyperboloid is $\mathrm{SL}(2, \mathbb{R})$

$$ds^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2.$$

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Strings on AdS_3

If the string propagates in a Lie group, $X: WS \to G$.
Think G as a symmetry, we have conserved currents.

$$[J^a_m, J^a_n] = k\delta^{ab}\delta_{m+n,0} + i f^{ab}{}_c J^c_{m+n}$$

Kac-Moody algebra.

States fall in representations $\mathfrak{g}$ and treat currents as oscillators.

Wess-Zumino-Witten model.

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Why? AdS/CFT correspondence.

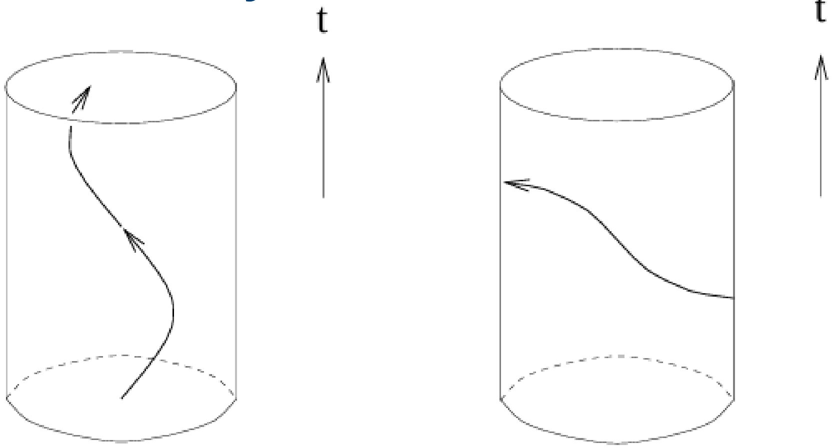
Strings on AdS_3

WZW action:

$$S = \frac{k}{8\pi\alpha'} \int d^2\sigma \mathrm{Tr}\left(g^{-1}\partial g g^{-1}\partial g\right) + k\Gamma_{WZ}$$

where the last term is the Wess-Zumino term.

Consider symmetries and classical geodesics:



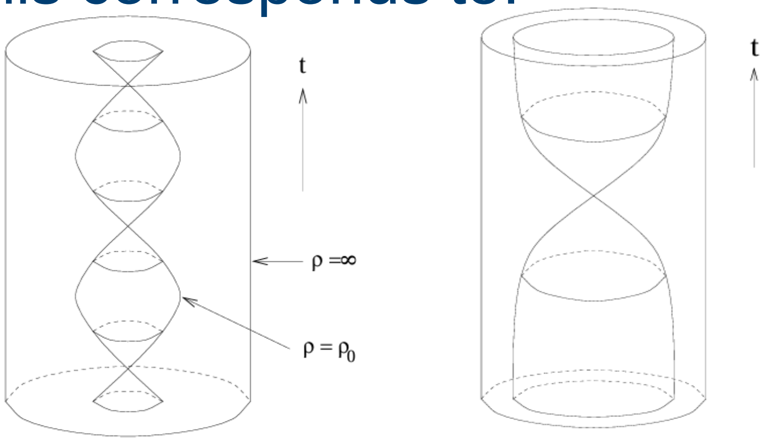
Strings on AdS_3

What about strings? Spectral flow:

Given a trajectory $g(\sigma,\tau)$, we can define new trajectories by the operation:

$$\sigma^w(g) = e^{-w\sigma^+ t^3} g(\sigma,\tau) e^{-w\sigma^- t^3}$$

This corresponds to: $t \to t + w\tau \quad \phi \to \phi + w\sigma$



$w \in Z$

Stretches along t, winds along ϕ

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Thank you for listening!