Algebraic Methods in Combinatorics (M16)

Mansur S. Boase

This course will cover a selection of the many beautiful applications of algebra in combinatorics, ranging from the 18th to the 21st century. The purpose is to introduce and motivate some of these methods and offer a foundation for what is a very active area of research.

We hope to cover the following material:

- **Enumerative Methods:** Counting with generating functions and determinants. Sums of powers formula. Introduction to Fibonacci, Bernoulli, Stirling and Catalan numbers. Matrix Tree theorem. Counting labelled and unlabelled trees.
- **Linear Algebra Methods:** Changing the space we are working in and using the rank. Lindström's theorem. Graham-Pollak theorem. Frankl-Wilson theorem. Applications to combinatorial geometry including the disproof of Borsuk's Conjecture. Skew-Bollobás theorem. Introduction to Shannon Capacity.
- **Polynomial Methods:** Dvir's solution to the Kakeya problem for finite fields. Alon's Combinatorial Nullstellensatz and applications to additive number theory, combinatorial geometry and list-colouring graphs. Tao's slice rank method building on work of Croot-Lev-Pach and Ellenberg-Gijswijt bounding capsets.

Prerequisites

None, although some familiarity with combinatorics and linear algebra would be helpful.

Literature

1. Alon, N., Combinatorial Nullstellensatz, *Combinatorics, Probability and Computing* **8**, 7–29 (1999). Available at https://www.tau.ac.il/~nogaa/PDFS/null2.pdf
2. Cameron, P.J., *Notes on Counting: An Introduction to Enumerative Combinatorics*, Cambridge University Press (2017).
3. Matoušek, J., *Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra*, Providence: American Mathematical Society (2010). Available at https://kam.mff.cuni.cz/~matousek/stml-53-matousek-1.pdf

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.