Random walks and phase transitions (L16)

Professor P.-F. Rodriguez

The course will focus on several topics related to critical phenomena in statistical physics models, that can be successfully tackled using probabilistic tools. In particular, certain models that exhibit intriguing phase transitions benefit from the rich interplay between random walks on the one hand, and percolation on the other. These models will be the main focus of this course.

A (non-exhaustive) selection of topics include:

- Random walks on graphs: Green's functions, potential theory, Harnack inequality, Dirichlet form, heat kernel bounds
- Gaussian free field (GFF): Markov Property, Gaussian techniques (orthogonal decompositions, Cameron-Martin theorem, entropy methods...)
- Percolation for the GFF: subcritical phase, supercritical phase, sharpness.

Prerequisites

This course assumes familiarity with probability (incl. measure theory) and analysis at Part II level, as well as the content of the Advanced Probability course. It is probably useful but not indispensable to follow the course "Random discrete structures" in Michaelmas.

Literature

Lecture notes will be provided during the course. Additional references are:

1. M. T. Barlow, *Random Walks and Heat Kernels on Graphs.* Cambridge University Press, 2017. Also available at:
https://www.cambridge.org/core/books/random-walks-and-heat-kernels-on-graphs/5B375D343025BCE91C682D49CDDB3A1A
2. G. Grimmett, *Probability on Graphs.* Cambridge University Press, 2010.
Also available at:
http://www.statslab.cam.ac.uk/~grg/books/pgs.html)
3. W. Werner and E. Powell, *Lecture notes on the Gaussian free field.* Cours Spécialisés S.M.F., 2022. Also available at:
https://arxiv.org/abs/2004.04720.
4. A.-S. Sznitman, *Topics in occupation times and the Gaussian free field.* Zurich Lectures in Advanced Mathematics, EMS Press, 2012. Also available at:
https://ethz.ch/content/dam/ethz/special-interest/math/department/Research/Research_Groups/Sznitman/Publications/2011_special_topics.pdf

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.