Random surfaces and probability in hyperbolic geometry (L16)

Non-Examinable (Graduate Level)

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Compact hyperbolic surfaces are the finite graphs of the continuous world. Indeed, while formally defined as "two-dimensional Riemannian manifolds of constant negative curvature", hyperbolic surfaces have geometric and analytic quantities analogous to many of those studied in graph theory. Often, the behaviour of a "typical" surface is in a way similar to that of a "typical" (regular) graph. Examples of such properties include:

1. the spectral gap of the Laplacian;
2. the Cheeger constant;
3. lengths of cycles;
4. decay of the heat kernel of the random walk / diffusion process.

In this course, we will explore randomness in and of compact hyperbolic surfaces. After covering the basic definitions and theorems concerning hyperbolic surfaces, we will discuss:

- the three popular ways to generate a random surface: The Brooks-Makover, Weil-Petersson, and random cover models (these are analogous to the configuration model, the "uniform measure" and the random cover model in graphs). We will focus mostly on the properties of the Brooks-Makover model;
- random processes on hyperbolic surfaces, such as Poisson point processes and Brownian loops;
- the small eigenvalues of a surface.

Prerequisites

The course covers topics at the intersection of probability, combinatorics, and (differential) geometry. Background in any of the above topics is useful; however, no advanced knowledge will be required in any of these fields (as long as you are willing to believe some things at face value).

Literature

1. Peter Buser. *Geometry and spectra of compact Riemann surfaces.* Modern Birkhäuser classics. 2010.
2. Laura Monk. *Geometry and Spectrum of typical hyperbolic surfaces.* Available at https://lauramonk.github.io/thesis.pdf.
3. Robert Brooks and Eran Makover. *Random Construction of Riemann Surfaces.* Available at https://projecteuclid.org/journals/journal-of-differential-geometry/volume-68/issue-1/Random-Construction-of-Riemann-Surfaces/10.4310/jdg/1102536712.full.