UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS   Part III

Friday, 8 June, 2012   9:00 am to 11:00 am

PAPER 24

COMPUTABLE FUNCTION THEORY

*Attempt no more than **THREE** questions.*
*There are **FOUR** questions in total.*
The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Exhibit a decidable partition of $[\mathbb{N}]^3$ (the set of triples from $\mathbb{N}$) that has no infinite decidable monochromatic set.

Exhibit a recursive perfect binary tree with no infinite recursive path.

2

Explain *function-in-intension* and *function-in-extension*.

What is a primitive recursive function? Is, for each i, the set of Gödel numbers of primitive recursive functions-in-intension of arity i decidable? By diagonalising over an enumeration of such Gödel numbers, or otherwise, exhibit a computable total function that is not primitive recursive.

3

Prove that there is a semidecidable set $X \subseteq \mathbb{N}$ with $\mathbb{N} \setminus X$ infinite such that X meets every infinite semidecidable set.

4

Explain many-one-reducibility and Turing-reducibility. Prove the theorem of Kleene and Post that there are incomparable Turing degrees below $0'$.

END OF PAPER