UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 10 June, 2014 9:00 am to 11:00 am

PAPER 48

ADVANCED STRING THEORY

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

The action I for the closed superstring in Minkowski spacetime is

$$I = -\int d^2\xi \ \det e_\mu^m \ \left(\frac{1}{2}\partial_\mu X^a \partial_\nu X_a h^{\mu\nu} + \frac{1}{2}\bar{\psi}^a \gamma^\mu \partial_\mu \psi_a - i\bar{\chi}_\mu \gamma^\nu \gamma^\mu \psi^a (\partial_\nu X_a - \frac{i}{4}\bar{\chi}_\nu \psi_a)\right)$$

where X^a are the coordinates of the superstring in spacetime, ψ^a are their superpartners, e_μ^m is the worldsheet zweibein, χ_μ is the worldsheet gravitino, $h_{\mu\nu}$ is the worldsheet metric and the worldsheet coordinates are ξ^μ.

Explain how one fixes the gauge and eliminates the unphysical states from the theory.

How does spacetime supersymmetry emerge?

Describe the massless states of the IIA theory.

Find the first level of massive physical states for the right moving sector only and demonstrate that the number of physical degrees of freedom in the R sector is the same as the number of physical degrees of freedom for the NS sector.

Explain how this is consistent with spacetime supersymmetry for the first massive level of the IIA closed superstring.

2

Write an essay describing how the superstring can be used to make a plausible "theory of everything."

3

The closed bosonic string can be coupled to a background metric g_{ab}, a Kalb–Ramond field B_{ab} and a dilaton Φ. The action for this string theory is then

$$I = \int d^2\xi \ \sqrt{-\det h} \ \left(\frac{1}{2}\partial_\mu X^a \partial_\nu X^b h^{\mu\nu} g_{ab} + \frac{1}{2}\partial_\mu X^a \partial_\nu X^b \epsilon^{\mu\nu} B_{ab} + {}^{(2)}R\,\Phi\right).$$

Relate the value of Φ to the string coupling constant.

Sketch the calculation of β_g to lowest order when $B_{ab} = \Phi = 0$.

Supposing that g_{ab} is independent of the spacetime co-ordinate y, and $B_{ab} = 0$, derive the T-duality Büscher rules for finding the dual metric g'_{ab} and Kalb–Ramond field B'_{ab} in terms of g_{ab}.

END OF PAPER