UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS   Part III

Thursday, 28 May, 2015   9:00 am to 12:00 pm

PAPER 43

QUANTUM FIELD THEORY

*Attempt no more than **THREE** questions.*
*There are **FOUR** questions in total.*
The questions carry equal weight.

1

Write an essay describing the path integral for a scalar field theory. It is expected that your essay will include some discussion of the Lagrangian and Hamiltonian formulation of scalar field theories, the path integral in quantum mechanics, the generalisation to field theory and how the introduction of source terms leads to a method of computing the expectation values of field operators.

2

In the theory of a massive real scalar field ϕ with an interaction Lagrangian $\frac{\lambda}{4!}\phi^4$, explain what the Feynman rules are.

Suppose that two scalar particles collide. In the centre-of-mass frame their energy is E. Find the differential scattering cross-section to lowest non-trivial order in λ.

3

The electromagnetic field is described by the Lagrangian

$$L = -\frac{1}{4}F_{ab}F^{ab},$$

where $F_{ab} = \partial_a A_b - \partial_b A_a$.

Describe how one proceeds to quantize this system using canonical methods in particular describing the nature of the field operator for the vector potential A_a. Carefully explain why the photon only has two polarization states.

Construct the Feynman propagator for the photon.

4

Starting from an action principle, derive the Dirac equation for the field Ψ.

How is the Dirac equation consistent with Lorentz invariance?

Suppose that the Dirac field has an electric charge q. Explain how to couple the electromagnetic field to the Dirac field in a way consistent with gauge invariance.

How does Ψ transform under an infinitesimal Lorentz tranformation?

Show that $\bar{\Psi}\Psi$ is a Lorentz scalar.

END OF PAPER